Mathematics (Course 18)

General Mathematics

18.01 Calculus
Prereq: None
U (Fall, Spring)
5-0-7 units. CALC I
Credit cannot also be received for 18.01A, ES.1801, ES.181A


Fall: L. Guth. Spring: Information: W. Minicozzi

18.01A Calculus
Prereq: Knowledge of differentiation and elementary integration
U (Fall; first half of term)
5-0-7 units. CALC I
Credit cannot also be received for 18.01, ES.1801, ES.181A

Six-week review of one-variable calculus, emphasizing material not on the high-school AB syllabus: integration techniques and applications, improper integrals, infinite series, applications to other topics, such as probability and statistics, as time permits. Prerequisites: one year of high-school calculus or the equivalent, with a score of 5 on the AB Calculus test (or the AB portion of the BC test, or an equivalent score on a standard international exam), or equivalent college transfer credit, or a passing grade on the first half of the 18.01 advanced standing exam.

D. Jerison

18.02 Calculus
Prereq: Calculus I (GIR)
U (Fall, Spring)
5-0-7 units. CALC II
Credit cannot also be received for 18.02, 18.02A, CC.1802, ES.1802, ES.182A

Calculus of several variables. Vector algebra in 3-space, determinants, matrices. Vector-valued functions of one variable, space motion. Scalar functions of several variables: partial differentiation, gradient, optimization techniques. Double integrals and line integrals in the plane; exact differentials and conservative fields; Green's theorem and applications, triple integrals, line and surface integrals in space, Divergence theorem, Stokes' theorem; applications.

Fall: S. Dyatlov. Spring: W. Minicozzi

18.02A Calculus
Prereq: Calculus I (GIR)
U (Fall, IAP, Spring; second half of term)
5-0-7 units. CALC II
Credit cannot also be received for 18.02, 18.02A, CC.1802, ES.1802, ES.182A

First half is taught during the last six weeks of the Fall term; covers material in the first half of 18.02 (through double integrals). Second half of 18.02A can be taken either during IAP (daily lectures) or during the second half of the Spring term; it covers the remaining material in 18.02.

Fall, IAP: J. W. M. Bush. Spring: W. Minicozzi

18.022 Calculus
Prereq: Calculus I (GIR)
U (Fall)
5-0-7 units. CALC II
Credit cannot also be received for 18.02, 18.02A, CC.1802, ES.1802, ES.182A

Calculus of several variables. Topics as in 18.02 but with more focus on mathematical concepts. Vector algebra, dot product, matrices, determinant. Functions of several variables, continuity, differentiability, derivative. Parametrized curves, arc length, curvature, torsion. Vector fields, gradient, curl, divergence. Multiple integrals, change of variables, line integrals, surface integrals. Stokes' theorem in one, two, and three dimensions.

G. Staffilani
18.03 Differential Equations
Prereq: None. Coreq: Calculus II (GIR)
U (Fall, Spring)
5-0-7 units. REST
Credit cannot also be received for 18.032, CC.1803
Fall: T. Collins. Spring: A. Lawrie

18.031 System Functions and the Laplace Transform
Prereq: None. Coreq: 18.03
U (IAP)
1-0-2 units
Studies basic continuous control theory as well as representation of functions in the complex frequency domain. Covers generalized functions, unit impulse response, and convolution; and Laplace transform, system (or transfer) function, and the pole diagram. Includes examples from mechanical and electrical engineering.
Information: H. R. Miller

18.032 Differential Equations
Prereq: None. Coreq: Calculus II (GIR)
U (Spring)
5-0-7 units. REST
Credit cannot also be received for 18.03, CC.1803
Covers much of the same material as 18.03 with more emphasis on theory. The point of view is rigorous and results are proven. Local existence and uniqueness of solutions.
Information: W. Minicozzi

18.04 Complex Variables with Applications
Prereq: Calculus II (GIR) and (18.03 or 18.032)
U (Spring)
4-0-8 units
Credit cannot also be received for 18.075, 18.0751
Complex algebra and functions; analyticity; contour integration, Cauchy's theorem; singularities, Taylor and Laurent series; residues, evaluation of integrals; multivalued functions, potential theory in two dimensions; Fourier analysis, Laplace transforms, and partial differential equations.
Y. Wang

18.05 Introduction to Probability and Statistics
Prereq: Calculus II (GIR)
U (Spring)
4-0-8 units. REST
J. Orloff

18.06 Linear Algebra
Prereq: Calculus II (GIR)
U (Fall, Spring)
4-0-8 units. REST
Credit cannot also be received for 18.700
Basic subject on matrix theory and linear algebra, emphasizing topics useful in other disciplines, including systems of equations, vector spaces, determinants, eigenvalues, singular value decomposition, and positive definite matrices. Applications to least-squares approximations, stability of differential equations, networks, Fourier transforms, and Markov processes. Uses linear algebra software. Compared with 18.700, more emphasis on matrix algorithms and many applications.
Fall: E. Mossel. Spring: A. Negut

18.062[J] Mathematics for Computer Science
Same subject as 6.042[J]
Prereq: Calculus I (GIR)
U (Fall, Spring)
5-0-7 units. REST
See description under subject 6.042[J].
Z. R. Abel, F. T. Leighton, A. Moitra

18.065 Matrix Methods in Data Analysis, Signal Processing, and Machine Learning
Subject meets with 18.0651
Prereq: 18.06
U (Spring)
3-0-9 units
Reviews linear algebra with applications to life sciences, finance, engineering, and big data. Covers singular value decomposition, weighted least squares, signal and image processing, principal component analysis, covariance and correlation matrices, directed and undirected graphs, matrix factorizations, neural nets, machine learning, and computations with large matrices.
G. Strang
18.0651 Matrix Methods in Data Analysis, Signal Processing, and Machine Learning
Subject meets with 18.065
Prereq: 18.06
G (Spring)
3-0-9 units
Reviews linear algebra with applications to life sciences, finance, engineering, and big data. Covers singular value decomposition, weighted least squares, signal and image processing, principal component analysis, covariance and correlation matrices, directed and undirected graphs, matrix factorizations, neural nets, machine learning, and computations with large matrices. Students in Course 18 must register for the undergraduate version, 18.065.
G. Strang

18.075 Methods for Scientists and Engineers
Subject meets with 18.0751
Prereq: Calculus II (GIR) and 18.03
U (Spring)
3-0-9 units
Credit cannot also be received for 18.04
Covers functions of a complex variable; calculus of residues. Includes ordinary differential equations; Bessel and Legendre functions; Sturm-Liouville theory; partial differential equations; heat equation; and wave equations.
H. Cheng

18.0751 Methods for Scientists and Engineers
Subject meets with 18.0751
Prereq: Calculus II (GIR) and 18.03
G (Spring)
3-0-9 units
Credit cannot also be received for 18.04
Covers functions of a complex variable; calculus of residues. Includes ordinary differential equations; Bessel and Legendre functions; Sturm-Liouville theory; partial differential equations; heat equation; and wave equations. Students in Courses 6, 8, 12, 18, and 22 must register for undergraduate version, 18.075.
H. Cheng

18.085 Computational Science and Engineering I
Subject meets with 18.0851
Prereq: Calculus II (GIR) and (18.03 or 18.032)
U (Fall, Spring, Summer)
3-0-9 units
Review of linear algebra, applications to networks, structures, and estimation, finite difference and finite element solution of differential equations, Laplace’s equation and potential flow, boundary-value problems, Fourier series, discrete Fourier transform, convolution. Frequent use of MATLAB in a wide range of scientific and engineering applications.
Fall: L. Demanet. Spring: L. Lu

18.0851 Computational Science and Engineering I
Subject meets with 18.085
Prereq: Calculus II (GIR) and (18.03 or 18.032)
G (Fall, Spring, Summer)
3-0-9 units
Review of linear algebra, applications to networks, structures, and estimation, finite difference and finite element solution of differential equations, Laplace’s equation and potential flow, boundary-value problems, Fourier series, discrete Fourier transform, convolution. Frequent use of MATLAB in a wide range of scientific and engineering applications. Students in Course 18 must register for the undergraduate version, 18.085.
Fall: L. Demanet. Spring: L. Lu

18.086 Computational Science and Engineering II
Subject meets with 18.0861
Prereq: Calculus II (GIR) and (18.03 or 18.032)
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: U (Spring)
3-0-9 units
Information: W. G. Strang
18.0861 Computational Science and Engineering II
Subject meets with 18.086
Prereq: Calculus II (GIR) and (18.03 or 18.032)
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units

Initial value problems: finite difference methods, accuracy and
stability, heat equation, wave equations, conservation laws and
shocks, level sets, Navier-Stokes. Solving large systems: elimination
with reordering, iterative methods, preconditioning, multigrid,
Krylov subspaces, conjugate gradients. Optimization and minimum
principles: weighted least squares, constraints, inverse problems,
calculus of variations, saddle point problems, linear programming,
duality, adjoint methods. Students in Course 18 must register for the
undergraduate version, 18.086.
Information: W. G. Strang

18.089 Review of Mathematics
Prereq: Permission of instructor
G (Summer)
5-0-7 units

One-week review of one-variable calculus (18.01), followed by
concentrated study covering multivariable calculus (18.02), two
hours per day for five weeks. Primarily for graduate students in
Course 2N. Degree credit allowed only in special circumstances.
Information: W. Minicozzi

18.094[J] Teaching College-Level Science and Engineering
Same subject as 1.95[J], 5.95[J], 7.59[J], 8.395[J]
Subject meets with 2.978
Prereq: None
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Fall)
2-0-2 units

See description under subject 5.95[J].
J. Rankin

18.095 Mathematics Lecture Series
Prereq: Calculus I (GIR)
U (IAP)
2-0-4 units
Can be repeated for credit.

Ten lectures by mathematics faculty members on interesting
topics from both classical and modern mathematics. All lectures
accessible to students with calculus background and an interest in
mathematics. At each lecture, reading and exercises are assigned.
Students prepare these for discussion in a weekly problem session.
Information: W. Minicozzi

18.098 Internship in Mathematics
Prereq: Permission of instructor
U (Fall, IAP, Spring, Summer)
Units arranged [P/D/F]
Can be repeated for credit.

Provides academic credit for students pursuing internships to gain
practical experience in the applications of mathematical concepts
and methods.
Information: W. Minicozzi

18.099 Independent Study
Prereq: Permission of instructor
U (Fall, IAP, Spring, Summer)
Units arranged
Can be repeated for credit.

Studies (during IAP) or special individual reading (during regular
terms). Arranged in consultation with individual faculty members
and subject to departmental approval. May not be used to satisfy
Mathematics major requirements.
Information: W. Minicozzi

Analysis

18.1001 Real Analysis
Prereq: Calculus II (GIR)
G (Fall, Spring)
3-0-9 units
Credit cannot also be received for 18.1002, 18.100A, 18.100B,
18.100P, 18.100Q

Covers fundamentals of mathematical analysis: convergence of
sequences and series, continuity, differentiability, Riemann integral,
sequences and series of functions, uniformity, interchange of limit
operations. Shows the utility of abstract concepts and teaches
understanding and construction of proofs. Proofs and definitions are
less abstract than in 18.100B. Gives applications where possible.
Concerned primarily with the real line. Students in Course 18 must
register for undergraduate version 18.100A.
Fall: C. Rodriguez. Spring: X. Yu
18.1002 Real Analysis
Subject meets with 18.100A, 18.100B
Prereq: Calculus II (GIR)
G (Fall, Spring)
3-0-9 units
Credit cannot also be received for 18.1001, 18.100P, 18.100Q

Covers fundamentals of mathematical analysis: convergence of sequences and series, continuity, differentiability, Riemann integral, sequences and series of functions, uniformity, interchange of limit operations. Shows the utility of abstract concepts and teaches understanding and construction of proofs. More demanding than 18.100A, for students with more mathematical maturity. Places more emphasis on point-set topology and n-space. Students in Course 18 must register for undergraduate version 18.100B. In person not required.

Fall: T. Colding. Spring: P-K Hung

18.100P Real Analysis
Prereq: Calculus II (GIR)
U (Spring)
4-0-11 units
Credit cannot also be received for 18.1001, 18.1002, 18.100A, 18.100B, 18.100Q

Covers fundamentals of mathematical analysis: convergence of sequences and series, continuity, differentiability, Riemann integral, sequences and series of functions, uniformity, interchange of limit operations. Shows the utility of abstract concepts and teaches understanding and construction of proofs. Proofs and definitions are less abstract than in 18.100B. Gives applications where possible. Concerned primarily with the real line. Includes instruction and practice in written communication. Enrollment limited.
B. Landon

18.100Q Real Analysis
Prereq: Calculus II (GIR)
U (Fall)
4-0-11 units
Credit cannot also be received for 18.1001, 18.1002, 18.100A, 18.100B, 18.100P

Covers fundamentals of mathematical analysis: convergence of sequences and series, continuity, differentiability, Riemann integral, sequences and series of functions, uniformity, interchange of limit operations. Shows the utility of abstract concepts and teaches understanding and construction of proofs. More demanding than 18.100A, for students with more mathematical maturity. Places more emphasis on point-set topology and n-space. Includes instruction and practice in written communication. Enrollment limited.
Y. Zhao

18.101 Analysis and Manifolds
Subject meets with 18.1011
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
U (Fall)
3-0-9 units

Introduction to the theory of manifolds: vector fields and densities on manifolds, integral calculus in the manifold setting and the manifold version of the divergence theorem. 18.901 helpful but not required.
R. B. Melrose
**18.101 Analysis and Manifolds**
Subject meets with 18.101
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
G (Fall)
3-0-9 units

Introduction to the theory of manifolds: vector fields and densities on manifolds, integral calculus in the manifold setting and the manifold version of the divergence theorem. 18.9011 helpful but not required. Students in Course 18 must register for the undergraduate version, 18.101.

*R. B. Melrose*

**18.102 Introduction to Functional Analysis**
Subject meets with 18.1021
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
U (Spring)
3-0-9 units


*R. B. Melrose*

**18.1021 Introduction to Functional Analysis**
Subject meets with 18.102
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
G (Spring)
3-0-9 units


Students in Course 18 must register for the undergraduate version, 18.102.

*R. B. Melrose*

**18.103 Fourier Analysis: Theory and Applications**
Subject meets with 18.103
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
G (Fall)
3-0-9 units

Roughly half the subject devoted to the theory of the Lebesgue integral with applications to probability, and half to Fourier series and Fourier integrals. Students in Course 18 must register for the undergraduate version, 18.103.

*A. Lawrie*

**18.104 Seminar in Analysis**
Prereq: 18.100A, 18.100B, 18.100P, or 18.100Q
U (Spring)
3-0-9 units

Students present and discuss material from books or journals. Topics vary from year to year. Instruction and practice in written and oral communication provided. Enrollment limited.

*G. Staffilani*

**18.112 Functions of a Complex Variable**
Subject meets with 18.1121
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
U (Fall)
3-0-9 units


*A. Borodin*
18.1121 Functions of a Complex Variable
Subject meets with 18.112
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
G (Fall)
3-0-9 units
A. Borodin

18.116 Riemann Surfaces
Prereq: 18.112
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Riemann surfaces, uniformization, Riemann-Roch Theorem. Theory of elliptic functions and modular forms. Some applications, such as to number theory.
T. S. Mrowka

18.117 Topics in Several Complex Variables
Prereq: 18.112 and 18.965
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Can be repeated for credit.
Harmonic theory on complex manifolds, Hodge decomposition theorem, Hard Lefschetz theorem. Vanishing theorems. Theory of Stein manifolds. As time permits students also study holomorphic vector bundles on Kahler manifolds.
B. Poonen

18.118 Topics in Analysis
Prereq: Permission of instructor
G (Fall)
Not offered regularly; consult department
3-0-9 units
Can be repeated for credit.
Topics vary from year to year.
L. Guth

18.125 Measure Theory and Analysis
Prereq: 18.100A, 18.100B, 18.100P, or 18.100Q
G (Spring)
3-0-9 units
Provides a rigorous introduction to Lebesgue’s theory of measure and integration. Covers material that is essential in analysis, probability theory, and differential geometry.
D. W. Stroock

18.137 Topics in Geometric Partial Differential Equations
Prereq: Permission of instructor
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Fall)
3-0-9 units
Can be repeated for credit.
Topics vary from year to year.
T. Colding

18.152 Introduction to Partial Differential Equations
Subject meets with 18.1521
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
U (Spring)
3-0-9 units
Introduces three main types of partial differential equations: diffusion, elliptic, and hyperbolic. Includes mathematical tools, real-world examples and applications, such as the Black-Scholes equation, the European options problem, water waves, scalar conservation laws, first order equations and traffic problems.
D. Jerison

18.1521 Introduction to Partial Differential Equations
Subject meets with 18.152
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
G (Spring)
3-0-9 units
Introduces three main types of partial differential equations: diffusion, elliptic, and hyperbolic. Includes mathematical tools, real-world examples and applications, such as the Black-Scholes equation, the European options problem, water waves, scalar conservation laws, first order equations and traffic problems. Students in Course 18 must register for the undergraduate version, 18.152.
D. Jerison
18.155 Differential Analysis I  
Prereq: 18.102 or 18.103  
G (Fall)  
3-0-9 units  
First part of a two-subject sequence. Review of Lebesgue integration.  
Spectral theorem, discrete and continuous spectrum. Homogeneous  
distributions. Fundamental solutions for elliptic, hyperbolic and  
parabolic differential operators. Recommended prerequisite: 18.112.  
In person not required.  
T. S. Mrowka

18.156 Differential Analysis II  
Prereq: 18.155  
G (Spring)  
3-0-9 units  
Second part of a two-subject sequence. Covers variable coefficient  
elliptic, parabolic and hyperbolic partial differential equations.  
L. Guth

18.157 Introduction to Microlocal Analysis  
Prereq: 18.155  
Acad Year 2020-2021: G (Spring)  
Acad Year 2021-2022: Not offered  
3-0-9 units  
The semi-classical theory of partial differential equations. Discussion  
of Pseudodifferential operators, Fourier integral operators,  
asymptotic solutions of partial differential equations, and the  
spectral theory of Schroedinger operators from the semi-classical  
perspective. Heavy emphasis placed on the symplectic geometric  
underpinnings of this subject.  
P. Hintz

18.158 Topics in Differential Equations  
Prereq: 18.157  
Acad Year 2020-2021: G (Spring)  
Acad Year 2021-2022: Not offered  
3-0-9 units  
Can be repeated for credit.  
Topics vary from year to year.  
R. B. Melrose

18.199 Graduate Analysis Seminar  
Prereq: Permission of instructor  
G (Fall)  
Not offered regularly; consult department  
3-0-9 units  
Can be repeated for credit.  
Studies original papers in differential analysis and differential  
equations. Intended for first- and second-year graduate students.  
Permission must be secured in advance.  
V. W. Guillemin

Discrete Applied Mathematics

18.200 Principles of Discrete Applied Mathematics  
Prereq: None. Coreq: 18.06  
U (Spring)  
4-0-11 units  
Credit cannot also be received for 18.200A  
Study of illustrative topics in discrete applied mathematics,  
including probability theory, information theory, coding theory,  
secret codes, generating functions, and linear programming.  
Instruction and practice in written communication provided.  
Enrollment limited.  
M. X. Goemans, D. Cifuentes

18.200A Principles of Discrete Applied Mathematics  
Prereq: None. Coreq: 18.06  
U (Fall)  
3-0-9 units  
Credit cannot also be received for 18.200  
Study of illustrative topics in discrete applied mathematics,  
including probability theory, information theory, coding theory,  
secret codes, generating functions, and linear programming.  
D. Cifuentes

18.204 Undergraduate Seminar in Discrete Mathematics  
Prereq: ((6.042[J]) or 18.200) and (18.06, 18.700, or 18.701) or  
permission of instructor  
U (Fall, Spring)  
3-0-9 units  
Seminar in combinatorics, graph theory, and discrete mathematics  
in general. Participants read and present papers from recent  
mathematics literature. Instruction and practice in written and oral  
communication provided. Enrollment limited.  
S. Dhara, N. Gadish, F. Gotti
18.211 Combinatorial Analysis  
Prereq: Calculus II (GIR) and (18.06, 18.700, or 18.701)  
U (Fall)  
3-0-9 units  
Combinatorial problems and methods for their solution.  
Enumeration, generating functions, recurrence relations,  
construction of bijections. Introduction to graph theory. Prior  
experience with abstraction and proofs is helpful.  
Z. Jiang

18.212 Algebraic Combinatorics  
Prereq: 18.701 or 18.703  
U (Spring)  
3-0-9 units  
Applications of algebra to combinatorics. Topics include walks  
in graphs, the Radon transform, groups acting on posets, Young  
tableaux, electrical networks.  
A. Postnikov

18.217 Combinatorial Theory  
Prereq: Permission of instructor  
G (Fall)  
3-0-9 units  
Can be repeated for credit.  
Content varies from year to year. In person not required.  
A. Postnikov

18.218 Topics in Combinatorics  
Prereq: Permission of instructor  
G (Spring)  
3-0-9 units  
Can be repeated for credit.  
Topics vary from year to year.  
E. Mossel

18.219 Seminar in Combinatorics  
Prereq: Permission of instructor  
G (Fall)  
Not offered regularly; consult department  
3-0-9 units  
Can be repeated for credit.  
Content varies from year to year. Readings from current research  
papers in combinatorics. Topics to be chosen and presented by the  
class.  
Information: Y. Zhao

18.225 Graph Theory and Additive Combinatorics (New)  
Prereq: ((18.701 or 18.703) and (18.100A, 18.100B, 18.100P, or  
18.100Q)) or permission of instructor  
Acad Year 2020-2021: Not offered  
Acad Year 2021-2022: G (Fall)  
3-0-9 units  
Introduction to extremal graph theory and additive combinatorics.  
Highlights common themes, such as the dichotomy between  
structure versus pseudorandomness. Topics include Turan-type  
problems, Szemeredi’s regularity lemma and applications,  
pseudorandom graphs, spectral graph theory, graph limits,  
arithmetic progressions (Roth, Szemeredi, Green-Tao), discrete  
Fourier analysis, Freiman’s theorem on sumsets and structure.  
Discusses current research topics and open problems.  
Y. Zhao

18.226 Probabilistic Methods in Combinatorics (New)  
Prereq: (18.211, 18.600, and (18.100A, 18.100B, 18.100P, or 18.100Q))  
or permission of instructor  
Acad Year 2020-2021: G (Fall)  
Acad Year 2021-2022: Not offered  
3-0-9 units  
Introduction to the probabilistic method, a fundamental and  
powerful technique in combinatorics and theoretical computer  
science. Focuses on methodology as well as combinatorial  
applications. Suitable for students with strong interest and  
background in mathematical problem solving. Topics include  
linearity of expectations, alteration, second moment, Lovasz local  
lemma, correlation inequalities, Janson inequalities, concentration  
inequalities, entropy method.  
Y. Zhao

Continuous Applied Mathematics

18.300 Principles of Continuum Applied Mathematics  
Prereq: Calculus II (GIR) and (18.03 or 18.032)  
U (Spring)  
3-0-9 units  
Covers fundamental concepts in continuous applied mathematics.  
Applications from traffic flow, fluids, elasticity, granular flows, etc.  
Also covers continuum limit; conservation laws, quasi-equilibrium;  
kinematic waves; characteristics, simple waves, shocks; diffusion  
(linear and nonlinear); numerical solution of wave equations;  
finite differences, consistency, stability; discrete and fast Fourier  
transforms; spectral methods; transforms and series (Fourier,  
Laplace). Additional topics may include sonic booms, Mach cone,  
caustics, lattices, dispersion and group velocity. Uses MATLAB  
computing environment.  
M. Durey
18.303 Linear Partial Differential Equations: Analysis and Numerics
Prereq: 18.06 or 18.700
U (Spring)
3-0-9 units
Provides students with the basic analytical and computational tools of linear partial differential equations (PDEs) for practical applications in science and engineering, including heat/diffusion, wave, and Poisson equations. Analytics emphasize the viewpoint of linear algebra and the analogy with finite matrix problems. Studies operator adjoints and eigenproblems, series solutions, Green’s functions, and separation of variables. Numerics focus on finite-difference and finite-element techniques to reduce PDEs to matrix problems, including stability and convergence analysis and implicit/explicit timestepping. Some programming required for homework and final project.
V. Heinonen

18.305 Advanced Analytic Methods in Science and Engineering
Prereq: 18.04, 18.075, or 18.112
G (Fall)
3-0-9 units
Covers expansion around singular points: the WKB method on ordinary and partial differential equations; the method of stationary phase and the saddle point method; the two-scale method and the method of renormalized perturbation; singular perturbation and boundary-layer techniques; WKB method on partial differential equations. In person not required.
H. Cheng

18.306 Advanced Partial Differential Equations with Applications
Prereq: (18.03 or 18.032) and (18.04, 18.075, or 18.112)
G (Spring)
3-0-9 units
R. R. Rosales

18.327 Topics in Applied Mathematics
Prereq: Permission of instructor
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Can be repeated for credit.
Topics vary from year to year.
L. Demanet

18.330 Introduction to Numerical Analysis
Prereq: Calculus II (GIR) and (18.03 or 18.032)
U (Spring)
3-0-9 units
Basic techniques for the efficient numerical solution of problems in science and engineering. Root finding, interpolation, approximation of functions, integration, differential equations, direct and iterative methods in linear algebra. Knowledge of programming in a language such as MATLAB, Python, or Julia is helpful.
D. Sanders

18.335[J] Introduction to Numerical Methods
Same subject as 6.337[J]
Prereq: 18.06, 18.700, or 18.701
G (Spring)
3-0-9 units
Advanced introduction to numerical analysis: accuracy and efficiency of numerical algorithms. In-depth coverage of sparse-matrix/iterative and dense-matrix algorithms in numerical linear algebra (for linear systems and eigenproblems). Floating-point arithmetic, backwards error analysis, conditioning, and stability. Other computational topics (e.g., numerical integration or nonlinear optimization) may also be surveyed. Final project involves some programming.
S. Johnson
Same subject as 6.335[J]  
Prereq: 6.336[J], 16.920[J], 18.085, 18.335[J], or permission of instructor  
G (Fall)  
3-0-9 units  

Unified introduction to the theory and practice of modern, near linear-time, numerical methods for large-scale partial-differential and integral equations. Topics include preconditioned iterative methods; generalized Fast Fourier Transform and other butterfly-based methods; multiresolution approaches, such as multigrid algorithms and hierarchical low-rank matrix decompositions; and low and high frequency Fast Multipole Methods. Example applications include aircraft design, cardiovascular system modeling, electronic structure computation, and tomographic imaging.  

K. Burns

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**18.337[J] Parallel Computing and Scientific Machine Learning**  
Same subject as 6.338[J]  
Prereq: 18.06, 18.700, or 18.701  
G (Fall)  
3-0-9 units  

Introduction to scientific machine learning with an emphasis on developing scalable differentiable programs. Covers scientific computing topics (numerical differential equations, dense and sparse linear algebra, Fourier transformations, parallelization of large-scale scientific simulation) simultaneously with modern data science (machine learning, deep neural networks, automatic differentiation), focusing on the emerging techniques at the connection between these areas, such as neural differential equations and physics-informed deep learning. Provides direct experience with the modern realities of optimizing code performance for supercomputers, GPUs, and multicores in a high-level language.  

C. Rackauckas

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**18.338 Eigenvalues of Random Matrices**  
Prereq: 18.701 or permission of instructor  
Acad Year 2020-2021: G (Fall)  
Acad Year 2021-2022: Not offered  
3-0-9 units  

Covers the modern main results of random matrix theory as it is currently applied in engineering and science. Topics include matrix calculus for finite and infinite matrices (e.g., Wigner’s semicircle and Marcenko-Pastur laws), free probability, random graphs, combinatorial methods, matrix statistics, stochastic operators, passage to the continuum limit, moment methods, and compressed sensing. Knowledge of MATLAB helpful, but not required.  

A. Edelman

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**18.352[J] Nonlinear Dynamics: The Natural Environment**  
Same subject as 12.009[J]  
Prereq: Calculus II (GIR) and Physics I (GIR); Coreq: 18.03  
Acad Year 2020-2021: Not offered  
Acad Year 2021-2022: U (Spring)  
3-0-9 units  

See description under subject 12.009[J].  
D. H. Rothman

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**18.353[J] Nonlinear Dynamics: Chaos**  
Same subject as 2.050[J], 12.006[J]  
Prereq: Physics II (GIR) and (18.03 or 18.032)  
U (Fall)  
3-0-9 units  

See description under subject 12.006[J].  
M. Durey

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**18.354[J] Nonlinear Dynamics: Continuum Systems**  
Same subject as 1.062[J], 12.207[J]  
Subject meets with 18.3541  
Prereq: Physics II (GIR) and (18.03 or 18.032)  
U (Spring)  
3-0-9 units  

General mathematical principles of continuum systems. From microscopic to macroscopic descriptions in the form of linear or nonlinear (partial) differential equations. Exact solutions, dimensional analysis, calculus of variations and singular perturbation methods. Stability, waves and pattern formation in continuum systems. Subject matter illustrated using natural fluid and solid systems found, for example, in geophysics and biology.  

O. Kodio

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**18.3541 Nonlinear Dynamics: Continuum Systems**  
Subject meets with 1.062[J], 12.207[J], 18.354[J]  
Prereq: Physics II (GIR) and (18.03 or 18.032)  
G (Spring)  
3-0-9 units  

General mathematical principles of continuum systems. From microscopic to macroscopic descriptions in the form of linear or nonlinear (partial) differential equations. Exact solutions, dimensional analysis, calculus of variations and singular perturbation methods. Stability, waves and pattern formation in continuum systems. Subject matter illustrated using natural fluid and solid systems found, for example, in geophysics and biology. Students in Courses 1, 12, and 18 must register for undergraduate version, 18.354[J].  
O. Kodio
**18.355 Fluid Mechanics**  
Prereq: 2.25, 12.800, or 18.354[J]  
Acad Year 2020-2021: Not offered  
Acad Year 2021-2022: G (Fall)  
3-0-9 units  

Topics include the development of Navier-Stokes equations, inviscid flows, boundary layers, lubrication theory, Stokes flows, and surface tension. Fundamental concepts illustrated through problems drawn from a variety of areas, including geophysics, biology, and the dynamics of sport. Particular emphasis on the interplay between dimensional analysis, scaling arguments, and theory. Includes classroom and laboratory demonstrations.  

J. W. Bush

**18.357 Interfacial Phenomena**  
Prereq: 2.25, 12.800, 18.354[J], 18.355, or permission of instructor  
Acad Year 2020-2021: G (Spring)  
Acad Year 2021-2022: Not offered  
3-0-9 units  

Fluid systems dominated by the influence of interfacial tension. Elucidates the roles of curvature pressure and Marangoni stress in a variety of hydrodynamic settings. Particular attention to drops and bubbles, soap films and minimal surfaces, wetting phenomena, water-repellency, surfactants, Marangoni flows, capillary origami and contact line dynamics. Theoretical developments are accompanied by classroom demonstrations. Highlights the role of surface tension in biology.  

J. W. Bush

**18.358[J] Nonlinear Dynamics and Turbulence**  
Same subject as 1.686[J], 2.033[J]  
Subject meets with 1.068  
Prereq: 1.060A  
Acad Year 2020-2021: Not offered  
Acad Year 2021-2022: G (Spring)  
3-2-7 units  

See description under subject 1.686[J].  

L. Bourouiba

**18.359[J] Mathematical Methods in Nanophotonics**  
Same subject as 8.315[J]  
Prereq: 8.07, 18.303, or permission of instructor  
Acad Year 2020-2021: Not offered  
Acad Year 2021-2022: G (Spring)  
3-0-9 units  

High-level approaches to understanding complex optical media, structured on the scale of the wavelength, that are not generally analytically solvable. The basis for understanding optical phenomena such as photonic crystals and band gaps, anomalous diffraction, mechanisms for optical confinement, optical fibers (new and old), nonlinearity, and integrated optical devices. Methods covered include linear algebra and eigensystems for Maxwell’s equations, symmetry groups and representation theory, Bloch’s theorem, numerical eigensolver methods, time and frequency-domain computation, perturbation theory, and coupled-mode theories.  

S. G. Johnson

**18.367 Waves and Imaging**  
Prereq: Permission of instructor  
Acad Year 2020-2021: G (Spring)  
Acad Year 2021-2022: Not offered  
3-0-9 units  

The mathematics of inverse problems involving waves, with examples taken from reflection seismology, synthetic aperture radar, and computerized tomography. Suitable for graduate students from all departments who have affinities with applied mathematics. Topics include acoustic, elastic, electromagnetic wave equations; geometrical optics; scattering series and inversion; migration and backprojection; adjoint-state methods; Radon and curvilinear Radon transforms; microlocal analysis of imaging; optimization, regularization, and sparse regression.  

L. Demanet

**18.369[J] Wave Propagation**  
Same subject as 1.138[J], 2.062[J]  
Prereq: 2.003[J] and 18.075  
Acad Year 2020-2021: G (Spring)  
Acad Year 2021-2022: Not offered  
3-0-9 units  

See description under subject 2.062[J].  

T. R. Akylas, R. R. Rosales
18.377[J] Nonlinear Dynamics and Waves
Same subject as 1.685[J], 2.034[J]
Prereq: Permission of instructor
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
A unified treatment of nonlinear oscillations and wave phenomena with applications to mechanical, optical, geophysical, fluid, electrical and flow-structure interaction problems. Nonlinear free and forced vibrations; nonlinear resonances; self-excited oscillations; lock-in phenomena. Nonlinear dispersive and nondispersive waves; resonant wave interactions; propagation of wave pulses and nonlinear Schrödinger equation. Nonlinear long waves and breaking; theory of characteristics; the Korteweg-de Vries equation; solitons and solitary wave interactions. Stability of shear flows. Some topics and applications may vary from year to year.
R. R. Rosales

18.384 Undergraduate Seminar in Physical Mathematics
Prereq: 12.006[J], 18.300, 18.354[J], or permission of instructor
U (Fall)
3-0-9 units
Covers the mathematical modeling of physical systems, with emphasis on the reading and presentation of papers. Addresses a broad range of topics, with particular focus on macroscopic physics and continuum systems: fluid dynamics, solid mechanics, and biophysics. Instruction and practice in written and oral communication provided. Enrollment limited.
O. Kodio

18.385[J] Nonlinear Dynamics and Chaos
Same subject as 2.036[J]
Prereq: 18.03 or 18.032
Acad Year 2020-2021: G (Fall)
Acad Year 2021-2022: Not offered
3-0-9 units
R. R. Rosales

18.397 Mathematical Methods in Physics
Prereq: 18.745 or some familiarity with Lie theory
G (Fall)
Not offered regularly; consult department
3-0-9 units
Can be repeated for credit.
Content varies from year to year. Recent developments in quantum field theory require mathematical techniques not usually covered in standard graduate subjects.
V. G. Kac

Theoretical Computer Science
18.400[J] Computability and Complexity Theory
Same subject as 6.045[J]
Prereq: 6.042[J]
U (Spring)
4-0-8 units
See description under subject 6.045[J].
R. Williams, R. Rubinfeld

18.404 Theory of Computation
Subject meets with 6.840[J], 18.4041[J]
Prereq: 6.042[J] or 18.200
U (Fall)
4-0-8 units
A more extensive and theoretical treatment of the material in 6.045[J]/18.400[J], emphasizing computability and computational complexity theory. Regular and context-free languages. Decidable and undecidable problems, reducibility, recursive function theory. Time and space measures on computation, completeness, hierarchy theorems, inherently complex problems, oracles, probabilistic computation, and interactive proof systems.
M. Sipser
18.404[J] Theory of Computation
Same subject as 6.840[J]
Subject meets with 18.404
Prereq: 6.042[J] or 18.200
G (Fall)
4-0-8 units
A more extensive and theoretical treatment of the material in
6.045[J]/18.400[J], emphasizing computability and computational
complexity theory. Regular and context-free languages. Decidable
and undecidable problems, reducibility, recursive function theory.
Time and space measures on computation, completeness, hierarchy
theorems, inherently complex problems, oracles, probabilistic
computation, and interactive proof systems. Students in Course 18
must register for the undergraduate version, 18.404.
M. Sipser

18.405[J] Advanced Complexity Theory
Same subject as 6.841[J]
Prereq: 18.404
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Fall)
3-0-9 units
Current research topics in computational complexity theory.
Nondeterministic, alternating, probabilistic, and parallel
computation models. Boolean circuits. Complexity classes and
complete sets. The polynomial-time hierarchy. Interactive proof
systems. Relativization. Definitions of randomness. Pseudorandomness and derandomizations. Interactive proof systems and
probabilistically checkable proofs.
R. Williams

18.408 Topics in Theoretical Computer Science
Prereq: Permission of instructor
G (Spring)
3-0-9 units
Can be repeated for credit.
Study of areas of current interest in theoretical computer science.
Topics vary from term to term.
A. Moitra, J. A. Kelner

18.410[J] Design and Analysis of Algorithms
Same subject as 6.046[J]
Prereq: 6.006
U (Fall, Spring)
4-0-8 units
See description under subject 6.046[J].
E. Demaine, M. Goemans

18.415[J] Advanced Algorithms
Same subject as 6.854[J]
Prereq: 6.046[J] and (6.042[J], 18.600, or 6.041)
G (Fall)
5-0-7 units
See description under subject 6.854[J].
A. Moitra, D. R. Karger

18.416[J] Randomized Algorithms
Same subject as 6.856[J]
Prereq: (6.041 or 6.042[J]) and (6.046[J] or 6.854[J])
Acad Year 2020-2021: G (Spring)
Acad Year 2021-2022: Not offered
5-0-7 units
See description under subject 6.856[J].
D. R. Karger

18.417 Introduction to Computational Molecular Biology
Prereq: 6.006, 6.01, or permission of instructor
G (Fall)
Not offered regularly; consult department
3-0-9 units
Introduces the basic computational methods used to model and
predict the structure of biomolecules (proteins, DNA, RNA). Covers
classical techniques in the field (molecular dynamics, Monte Carlo,
dynamic programming) to more recent advances in analyzing and
predicting RNA and protein structure, ranging from Hidden Markov
Models and 3-D lattice models to attribute Grammars and tree
Grammars.
Information: B. Berger

18.418[J] Topics in Computational Molecular Biology
Same subject as HST.504[J]
Prereq: 6.047, 18.417, or permission of instructor
G (Spring)
3-0-9 units
Can be repeated for credit.
Covers current research topics in computational molecular biology.
Recent research papers presented from leading conferences such as
the International Conference on Computational Molecular Biology
(RECOMB) and the Conference on Intelligent Systems for Molecular
Biology (ISMB). Topics include original research (both theoretical
and experimental) in comparative genomics, sequence and structure
analysis, molecular evolution, proteomics, gene expression,
transcriptional regulation, biological networks, drug discovery,
and privacy. Recent research by course participants also covered.
Participants will be expected to present individual projects to the
class.
B. Berger
18.424 Seminar in Information Theory
Prereq: (6.041, 18.05, or 18.600) and (18.06, 18.700, or 18.701)
U (Spring)
3-0-9 units
Considers various topics in information theory, including data compression, Shannon's Theorems, and error-correcting codes. Students present and discuss the subject matter. Instruction and practice in written and oral communication provided. Enrollment limited.

P. W. Shor

18.425[J] Cryptography and Cryptanalysis
Same subject as 6.875[J]
Prereq: 6.046[J]
G (Fall)
3-0-9 units
See description under subject 6.875[J].

S. Goldwasser, S. Micali, V. Vaikuntanathan

18.434 Seminar in Theoretical Computer Science
Prereq: 6.046[J]
U (Fall)
3-0-9 units
Topics vary from year to year. Students present and discuss the subject matter. Instruction and practice in written and oral communication provided. Enrollment limited.

J. Gaudio

18.435[J] Quantum Computation
Same subject as 2.111[J], 8.370[J]
Prereq: Permission of instructor
G (Fall)
3-0-9 units
Provides an introduction to the theory and practice of quantum computation. Topics covered: physics of information processing; quantum algorithms including the factoring algorithm and Grover’s search algorithm; quantum error correction; quantum communication and cryptography. Knowledge of quantum mechanics helpful but not required.

I. Chuang, A. Harrow, S. Lloyd, P. Shor

18.436[J] Quantum Information Science
Same subject as 6.443[J], 8.371[J]
Prereq: 18.435[J]
G (Spring)
3-0-9 units
See description under subject 8.371[J].

I. Chuang, A. Harrow

18.437[J] Distributed Algorithms
Same subject as 6.852[J]
Prereq: 6.046[J]
Acad Year 2020-2021: G (Fall)
Acad Year 2021-2022: Not offered
3-0-9 units
See description under subject 6.852[J].

N. A. Lynch

18.453 Combinatorial Optimization
Subject meets with 18.4531
Prereq: 18.06, 18.700, or 18.701
Acad Year 2020-2021: U (Spring)
Acad Year 2021-2022: Not offered
3-0-9 units
Thorough treatment of linear programming and combinatorial optimization. Topics include matching theory, network flow, matroid optimization, and how to deal with NP-hard optimization problems. Prior exposure to discrete mathematics (such as 18.200) helpful.

Information: W. C. Franks

18.4531 Combinatorial Optimization
Subject meets with 18.453
Prereq: 18.06, 18.700, or 18.701
Acad Year 2020-2021: G (Spring)
Acad Year 2021-2022: Not offered
3-0-9 units
Thorough treatment of linear programming and combinatorial optimization. Topics include matching theory, network flow, matroid optimization, and how to deal with NP-hard optimization problems. Prior exposure to discrete mathematics (such as 18.200) helpful. Students in Course 18 must register for the undergraduate version, 18.453.

W. C. Franks

18.455 Advanced Combinatorial Optimization
Prereq: 18.453 or permission of instructor
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Advanced treatment of combinatorial optimization with an emphasis on combinatorial aspects. Non-bipartite matchings, submodular functions, matroid intersection/union, matroid matching, submodular flows, multicommodity flows, packing and connectivity problems, and other recent developments.

M. X. Goemans
18.456[J] Algebraic Techniques and Semidefinite Optimization
Same subject as 6.256[J]
Prereq: 6.251[J] or 15.093[J]
Acad Year 2020-2021: G (Spring)
Acad Year 2021-2022: Not offered
3-0-9 units
See description under subject 6.256[J].
P. Parrilo

Logic

18.504 Seminar in Logic
Prereq: (18.06, 18.510, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
Acad Year 2020-2021: U (Fall)
Acad Year 2021-2022: Not offered
3-0-9 units
Students present and discuss the subject matter taken from current journals or books. Topics vary from year to year. Instruction and practice in written and oral communication provided. Enrollment limited.
H. Cohn

18.510 Introduction to Mathematical Logic and Set Theory
Prereq: None
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: U (Fall)
3-0-9 units
H. Cohn

18.515 Mathematical Logic
Prereq: Permission of instructor
G (Spring)
Not offered regularly; consult department
3-0-9 units
Information: B. Poonen

Probability and Statistics

18.600 Probability and Random Variables
Prereq: Calculus II (GIR)
U (Fall, Spring)
4-0-8 units. REST
Credit cannot also be received for 6.041, 6.431, 15.079, 15.0791
Probability spaces, random variables, distribution functions. Binomial, geometric, hypergeometric, Poisson distributions. Uniform, exponential, normal, gamma and beta distributions. Conditional probability, Bayes theorem, joint distributions. Chebyshev inequality, law of large numbers, and central limit theorem. Credit cannot also be received for 6.041A or 6.041B.
J. A. Kelner, S. Sheffield

18.615 Introduction to Stochastic Processes
Prereq: 6.041 or 18.600
G (Spring)
3-0-9 units
E. Mossel

18.642 Topics in Mathematics with Applications in Finance
Prereq: 18.03, 18.06, and (18.05 or 18.600)
U (Fall)
3-0-9 units
Introduction to mathematical concepts and techniques used in finance. Lectures focusing on linear algebra, probability, statistics, stochastic processes, and numerical methods are interspersed with lectures by financial sector professionals illustrating the corresponding application in the industry. Prior knowledge of economics or finance helpful but not required.
P. Kempthorne, V. Strela, J. Xia
Same subject as IDS.014[J]
Subject meets with 18.6501
Prereq: 6.041 or 18.600
U (Fall, Spring)
4-0-8 units
Credit cannot also be received for 15.075[J], IDS.013[J]
In-depth introduction to the theoretical foundations of statistical methods that are useful in many applications. Enables students to understand the role of mathematics in the research and development of efficient statistical methods. Topics include methods for estimation (maximum likelihood estimation, method of moments, M-estimation), hypothesis testing (Wald’s test, likelihood ratio test, T tests, goodness of fit), Bayesian statistics, linear regression, generalized linear models, and principal component analysis.
Fall: P. Rigollet. Spring: T. Maunu

18.6501 Fundamentals of Statistics
Subject meets with 18.650[J], IDS.014[J]
Prereq: 6.041 or 18.600
G (Fall, Spring)
4-0-8 units
Credit cannot also be received for 15.075[J], IDS.013[J]
In-depth introduction to the theoretical foundations of statistical methods that are useful in many applications. Enables students to understand the role of mathematics in the research and development of efficient statistical methods. Topics include methods for estimation (maximum likelihood estimation, method of moments, M-estimation), hypothesis testing (Wald’s test, likelihood ratio test, T tests, goodness of fit), Bayesian statistics, linear regression, generalized linear models, and principal component analysis.
Fall: P. Rigollet. Spring: T. Maunu

18.655 Mathematical Statistics
Prereq: (18.650[J] and (18.100A, 18.100A, 18.100P, or 18.100Q)) or permission of instructor
G (Fall)
3-0-9 units
Decision theory, estimation, confidence intervals, hypothesis testing. Introduces large sample theory. Asymptotic efficiency of estimates. Exponential families. Sequential analysis. Prior exposure to both probability and statistics at the university level is assumed.
T. Maunu

Same subject as 9.521[J], IDS.160[J]
Prereq: (6.436[J], 18.06, and 18.6501) or permission of instructor
G (Spring)
3-0-9 units
See description under subject 9.521[J].
S. Rakhlin, P. Rigollet

18.657 Topics in Statistics
Prereq: Permission of instructor
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Can be repeated for credit.
Topics vary from term to term.
P. Rigollet

18.675 Theory of Probability
Prereq: 18.100A, 18.100B, 18.100P, or 18.100Q
G (Fall)
3-0-9 units
Sums of independent random variables, central limit phenomena, infinitely divisible laws, Levy processes, Brownian motion, conditioning, and martingales. Prior exposure to probability (e.g., 18.600) recommended.
Y. Wang

18.676 Stochastic Calculus
Prereq: 18.175
G (Spring)
3-0-9 units
Introduction to stochastic processes, building on the fundamental example of Brownian motion. Topics include Brownian motion, continuous parameter martingales, Ito’s theory of stochastic differential equations, Markov processes and partial differential equations, and may also include local time and excursion theory. Students should have familiarity with Lebesgue integration and its application to probability.
N. Sun

18.677 Topics in Stochastic Processes
Prereq: 18.675
G (Spring)
3-0-9 units
Can be repeated for credit.
Topics vary from year to year.
A. Borodin
Algebra and Number Theory

18.700 Linear Algebra
Prereq: Calculus II (GIR)
U (Fall)
3-0-9 units. REST
Credit cannot also be received for 18.06
Vector spaces, systems of linear equations, bases, linear
independence, matrices, determinants, eigenvalues, inner products,
quadratic forms, and canonical forms of matrices. More emphasis on
theory and proofs than in 18.06.
V. G. Kac

18.701 Algebra I
Prereq: 18.100A, 18.100B, 18.100P, 18.100Q, or permission of
instructor
U (Fall)
3-0-9 units
18.701-18.702 is more extensive and theoretical than the
18.700-18.703 sequence. Experience with proofs necessary. 18.701
focuses on group theory, geometry, and linear algebra.
B. Poonen

18.702 Algebra II
Prereq: 18.701
U (Spring)
3-0-9 units
Continuation of 18.701. Focuses on group representations, rings,
ideals, fields, polynomial rings, modules, factorization, integers in
quadratic number fields, field extensions, and Galois theory.
M. Artin

18.703 Modern Algebra
Prereq: Calculus II (GIR)
U (Spring)
3-0-9 units
Focuses on traditional algebra topics that have found greatest
application in science and engineering as well as in mathematics:
group theory, emphasizing finite groups; ring theory, including
ideals and unique factorization in polynomial and Euclidean rings;
field theory, including properties and applications of finite fields.
18.700 and 18.703 together form a standard algebra sequence.
V. G. Kac

18.704 Seminar in Algebra
Prereq: 18.701, (18.06 and 18.703), or (18.700 and 18.703)
U (Fall, Spring)
3-0-9 units
Topics vary from year to year. Students present and discuss
the subject matter. Instruction and practice in written and oral
communication provided. Some experience with proofs required.
Enrollment limited.
Fall: C. Chan. Spring: J. Wang

18.705 Commutative Algebra
Prereq: 18.702
G (Fall)
3-0-9 units
Exactness, direct limits, tensor products, Cayley-Hamilton theorem,
integral dependence, localization, Cohen-Seidenberg theory,
Noether normalization, Nullstellensatz, chain conditions, primary
decomposition, length, Hilbert functions, dimension theory,
completion, Dedekind domains.
R. Bezrukavnikov

18.706 Noncommutative Algebra
Prereq: 18.702
Acad Year 2020-2021: G (Fall)
Acad Year 2021-2022: Not offered
3-0-9 units
Topics may include Wedderburn theory and structure of Artinian
rings, Morita equivalence and elements of category theory,
localization and Goldie’s theorem, central simple algebras and the
Brauer group, representations, polynomial identity rings, invariant
time growth of algebras, Gelfand-Kirillov dimension.
Z. Yun

18.708 Topics in Algebra
Prereq: 18.705
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Can be repeated for credit.
Topics vary from year to year.
Z. Yun
18.715 Introduction to Representation Theory
Prereq: 18.702 or 18.703
Acad Year 2020-2021: G (Fall)
Acad Year 2021-2022: Not offered
3-0-9 units
Algebras, representations, Schur's lemma. Representations of SL(2).
Representations of finite groups, Maschke's theorem, characters,
applications. Induced representations, Burnside's theorem, Mackey
formula, Frobenius reciprocity. Representations of quivers.
G. Lusztig

18.721 Introduction to Algebraic Geometry
Prereq: 18.702 and 18.901
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: U (Spring)
3-0-9 units
Presents basic examples of complex algebraic varieties, affine and
projective algebraic geometry, sheaves, cohomology.
M. Artin

18.725 Algebraic Geometry I
Prereq: None. Coreq: 18.705
G (Fall)
3-0-9 units
Introduces the basic notions and techniques of modern algebraic
geometry. Covers fundamental notions and results about algebraic
varieties over an algebraically closed field; relations between
complex algebraic varieties and complex analytic varieties;
and examples with emphasis on algebraic curves and surfaces.
Introduction to the language of schemes and properties of
morphisms. Knowledge of elementary algebraic topology,
elementary differential geometry recommended, but not required.
D. Maulik

18.726 Algebraic Geometry II
Prereq: 18.725
G (Spring)
3-0-9 units
Continuation of the introduction to algebraic geometry given in
18.725. More advanced properties of the varieties and morphisms of
schemes, as well as sheaf cohomology.
D. Maulik

18.727 Topics in Algebraic Geometry
Prereq: 18.725
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Can be repeated for credit.
Topics vary from year to year.
A. Negut

18.737 Algebraic Groups
Prereq: 18.705
Acad Year 2020-2021: G (Spring)
Acad Year 2021-2022: Not offered
3-0-9 units
Structure of linear algebraic groups over an algebraically closed
field, with emphasis on reductive groups. Representations of groups
over a finite field using methods from etale cohomology. Some
results from algebraic geometry are stated without proof.
B. Poonen

18.745 Lie Groups and Lie Algebras I
Prereq: (18.701 or 18.703) and (18.100A, 18.100B, 18.100P, or
18.100Q)
G (Fall)
3-0-9 units
Covers fundamentals of the theory of Lie algebras and related
groups. Topics may include theorems of Engel and Lie; enveloping
algebra, Poincare-Birkhoff-Witt theorem; classification and
construction of semisimple Lie algebras; the center of their
enveloping algebras; elements of representation theory; compact Lie
groups and/or finite Chevalley groups. In person not required.
P. I. Etingof

18.747 Infinite-dimensional Lie Algebras
Prereq: 18.745
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Fall)
3-0-9 units
Topics vary from year to year.
P. I. Etingof

18.748 Topics in Lie Theory
Prereq: Permission of instructor
Acad Year 2020-2021: G (Fall)
Acad Year 2021-2022: Not offered
3-0-9 units
Can be repeated for credit.
Topics vary from year to year. In person not required.
J-L. Kim
18.755 Lie Groups and Lie Algebras II
Prereq: 18.745 or permission of instructor
G (Spring)
3-0-9 units
A more in-depth treatment of Lie groups and Lie algebras. Topics may include homogeneous spaces and groups of automorphisms; representations of compact groups and their geometric realizations, Peter-Weyl theorem; invariant differential forms and cohomology of Lie groups and homogeneous spaces; complex reductive Lie groups, classification of real reductive groups.
P. I. Etingof

18.757 Representations of Lie Groups
Prereq: 18.745 or 18.755
G (Spring)
Not offered regularly; consult department
3-0-9 units
Covers representations of locally compact groups, with emphasis on compact groups and abelian groups. Includes Peter-Weyl theorem and Cartan-Weyl highest weight theory for compact Lie groups.
Information: R. Bezrukavnikov

18.781 Theory of Numbers
Prereq: None
U (Spring)
3-0-9 units
An elementary introduction to number theory with no algebraic prerequisites. Primes, congruences, quadratic reciprocity, diophantine equations, irrational numbers, continued fractions, partitions.
J-L Kim

18.782 Introduction to Arithmetic Geometry
Prereq: 18.702
Acad Year 2020-2021: U (Fall)
Acad Year 2021-2022: Not offered
3-0-9 units
Exposes students to arithmetic geometry, motivated by the problem of finding rational points on curves. Includes an introduction to p-adic numbers and some fundamental results from number theory and algebraic geometry, such as the Hasse-Minkowski theorem and the Riemann-Roch theorem for curves. Additional topics may include Mordell’s theorem, the Weil conjectures, and Jacobian varieties.
D. Roe

18.783 Elliptic Curves
Subject meets with 18.7831
Prereq: 18.702, 18.703, or permission of instructor
Acad Year 2020-2021: U (Spring)
Acad Year 2021-2022: Not offered
3-0-9 units
Computationally focused introduction to elliptic curves, with applications to number theory and cryptography. Topics include point-counting, isogenies, pairings, and the theory of complex multiplication, with applications to integer factorization, primality proving, and elliptic curve cryptography. Includes a brief introduction to modular curves and the proof of Fermat’s Last Theorem.
A. Sutherland

18.7831 Elliptic Curves
Subject meets with 18.783
Prereq: 18.702, 18.703, or permission of instructor
Acad Year 2020-2021: G (Spring)
Acad Year 2021-2022: Not offered
3-0-9 units
Computationally focused introduction to elliptic curves, with applications to number theory and cryptography. Topics include point-counting, isogenies, pairings, and the theory of complex multiplication, with applications to integer factorization, primality proving, and elliptic curve cryptography. Includes a brief introduction to modular curves and the proof of Fermat’s Last Theorem. Students in Course 18 must register for the undergraduate version, 18.783.
A. Sutherland

18.784 Seminar in Number Theory
Prereq: 18.701 or (18.703 and (18.06 or 18.700))
U (Fall)
3-0-9 units
Topics vary from year to year. Students present and discuss the subject matter. Instruction and practice in written and oral communication provided. In person not required. Enrollment limited.
D. Kriz

18.785 Number Theory I
Prereq: None. Coreq: 18.705
G (Fall)
3-0-9 units
Dedekind domains, unique factorization of ideals, splitting of primes. Lattice methods, finiteness of the class group, Dirichlet’s unit theorem. Local fields, ramification, discriminants. Zeta and L-functions, analytic class number formula. Adeles and ideles. Statements of class field theory and the Chebotarev density theorem.
W. Zhang
18.786 Number Theory II
Prereq: 18.785
G (Spring)
3-0-9 units
Continuation of 18.785. More advanced topics in number theory, such as Galois cohomology, proofs of class field theory, modular forms and automorphic forms, Galois representations, or quadratic forms.
W. Zhang

18.787 Topics in Number Theory
Prereq: Permission of instructor
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Fall)
3-0-9 units
Can be repeated for credit.
Topics vary from year to year.
W. Zhang

Mathematics Laboratory

18.821 Project Laboratory in Mathematics
Prereq: Two mathematics subjects numbered 18.10 or above
U (Fall, Spring)
3-6-3 units. Institute LAB
Guided research in mathematics, employing the scientific method. Students confront puzzling and complex mathematical situations, through the acquisition of data by computer, pencil and paper, or physical experimentation, and attempt to explain them mathematically. Students choose three projects from a large collection of options. Each project results in a laboratory report subject to revision; oral presentation on one or two projects. Projects drawn from many areas, including dynamical systems, number theory, algebra, fluid mechanics, asymptotic analysis, knot theory, and probability. In person not required. Enrollment limited.
A. Negut, Z. Yun

Topology and Geometry

18.900 Geometry and Topology in the Plane
Prereq: 18.03 or 18.06
U (Fall)
3-0-9 units
Covers selected topics in geometry and topology, which can be visualized in the two-dimensional plane. Polygons and polygonal paths. Billiards. Closed curves and immersed curves. Algebraic curves. Triangulations and complexes. Hyperbolic geometry. Geodesics and curvature. Other topics may be included as time permits.
P. Seidel

18.901 Introduction to Topology
Subject meets with 18.9011
Prereq: 18.100A, 18.100B, 18.100P, 18.100Q, or permission of instructor
U (Fall, Spring)
3-0-9 units
Introduces topology, covering topics fundamental to modern analysis and geometry. Topological spaces and continuous functions, connectedness, compactness, separation axioms, covering spaces, and the fundamental group.
Fall: I. Dai. Spring: G. Lusztig

18.9011 Introduction to Topology
Subject meets with 18.901
Prereq: 18.100A, 18.100B, 18.100P, 18.100Q, or permission of instructor
G (Fall, Spring)
3-0-9 units
Introduces topology, covering topics fundamental to modern analysis and geometry. Topological spaces and continuous functions, connectedness, compactness, separation axioms, covering spaces, and the fundamental group. Students in Course 18 must register for the undergraduate version, 18.901.
Fall: I. Dai. Spring: G. Lusztig

18.904 Seminar in Topology
Prereq: 18.901
U (Spring)
3-0-9 units
Topics vary from year to year. Students present and discuss the subject matter. Instruction and practice in written and oral communication provided. Enrollment limited.
L. Piccirillo
18.905 Algebraic Topology I  
Prereq: 18.901 and (18.701 or 18.703)  
G (Fall)  
3-0-9 units  
Singular homology, CW complexes, universal coefficient and Künneth theorems, cohomology, cup products, Poincaré duality.  
*J. Hahn*

18.906 Algebraic Topology II  
Prereq: 18.905  
G (Spring)  
3-0-9 units  
Continues the introduction to Algebraic Topology from 18.905. Topics include basic homotopy theory, spectral sequences, characteristic classes, and cohomology operations.  
*P. Seidel*

18.917 Topics in Algebraic Topology  
Prereq: 18.906  
Acad Year 2020-2021: Not offered  
Acad Year 2021-2022: G (Spring)  
3-0-9 units  
Can be repeated for credit.  
Content varies from year to year. Introduces new and significant developments in algebraic topology with the focus on homotopy theory and related areas.  
*Information: H. R. Miller*

18.919 Graduate Topology Seminar  
Prereq: 18.906  
G (Fall)  
3-0-9 units  
Study and discussion of important original papers in the various parts of algebraic topology. Open to all students who have taken 18.906 or the equivalent, not only prospective topologists.  
*H. R. Miller*

18.937 Topics in Geometric Topology  
Prereq: Permission of instructor  
Acad Year 2020-2021: G (Spring)  
Acad Year 2021-2022: Not offered  
3-0-9 units  
Can be repeated for credit.  
Content varies from year to year. Introduces new and significant developments in geometric topology.  
*T. S. Mrowka*

18.950 Differential Geometry  
Subject meets with 18.9501  
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)  
U (Spring)  
3-0-9 units  
Introduction to differential geometry, centered on notions of curvature. Starts with curves in the plane, and proceeds to higher dimensional submanifolds. Computations in coordinate charts: first and second fundamental form, Christoffel symbols. Discusses the distinction between extrinsic and intrinsic aspects, in particular Gauss' theorema egregium. The Gauss-Bonnet theorem. Geodesics. Examples such as hyperbolic space.  
*T. Collins*

18.9501 Differential Geometry  
Subject meets with 18.950  
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)  
G (Spring)  
3-0-9 units  
Introduction to differential geometry, centered on notions of curvature. Starts with curves in the plane, and proceeds to higher dimensional submanifolds. Computations in coordinate charts: first and second fundamental form, Christoffel symbols. Discusses the distinction between extrinsic and intrinsic aspects, in particular Gauss' theorema egregium. The Gauss-Bonnet theorem. Geodesics. Examples such as hyperbolic space. Students in Course 18 must register for the undergraduate version, 18.950.  
*T. Collins*

18.952 Theory of Differential Forms  
Prereq: 18.101 and (18.700 or 18.701)  
U (Spring)  
3-0-9 units  
Multilinear algebra: tensors and exterior forms. Differential forms on \( \mathbb{R}^n \): exterior differentiation, the pull-back operation and the Poincaré lemma. Applications to physics: Maxwell's equations from the differential form perspective. Integration of forms on open sets of \( \mathbb{R}^n \). The change of variables formula revisited. The degree of a differentiable mapping. Differential forms on manifolds and De Rham theory. Integration of forms on manifolds and Stokes' theorem. The push-forward operation for forms. Thom forms and intersection theory. Applications to differential topology.  
*V. W. Guillemin*
18.965 Geometry of Manifolds I
Prereq: 18.101, 18.950, or 18.952
G (Fall)
3-0-9 units
Differential forms, introduction to Lie groups, the DeRham theorem, Riemannian manifolds, curvature, the Hodge theory. 18.965 is a continuation of 18.965 and focuses more deeply on various aspects of the geometry of manifolds. Contents vary from year to year, and can range from Riemannian geometry (curvature, holonomy) to symplectic geometry, complex geometry and Hodge-Kahler theory, or smooth manifold topology. Prior exposure to calculus on manifolds, as in 18.952, recommended. In person not required.

W. Minicozzi

18.966 Geometry of Manifolds II
Prereq: 18.965
G (Spring)
3-0-9 units
Continuation of 18.965, focusing more deeply on various aspects of the geometry of manifolds. Contents vary from year to year, and can range from Riemannian geometry (curvature, holonomy) to symplectic geometry, complex geometry and Hodge-Kahler theory, or smooth manifold topology.

T. Colding

18.968 Topics in Geometry
Prereq: 18.965
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Can be repeated for credit.

Content varies from year to year.

P. Seidel

18.979 Graduate Geometry Seminar
Prereq: Permission of instructor
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: G (Spring)
3-0-9 units
Can be repeated for credit.

Content varies from year to year. Study of classical papers in geometry and in applications of analysis to geometry and topology.

T. Mrowka

18.994 Seminar in Geometry
Prereq: (18.06, 18.700, or 18.701) and (18.100A, 18.100B, 18.100P, or 18.100Q)
Acad Year 2020-2021: Not offered
Acad Year 2021-2022: U (Fall)
3-0-9 units
Students present and discuss subject matter taken from current journals or books. Topics vary from year to year. Instruction and practice in written and oral communication provided. Enrollment limited.

Information: W. Minicozzi

18.999 Research in Mathematics
Prereq: Permission of instructor
G (Fall, Spring, Summer)
Units arranged
Can be repeated for credit.

Opportunity for study of graduate-level topics in mathematics under the supervision of a member of the department. For graduate students desiring advanced work not provided in regular subjects.

Information: W. Minicozzi

18.UR Undergraduate Research
Prereq: Permission of instructor
U (Fall, IAP, Spring, Summer)
Units arranged [P/D/F]
Can be repeated for credit.

Undergraduate research opportunities in mathematics. Permission required in advance to register for this subject. For further information, consult the departmental coordinator.

Information: W. Minicozzi

18.THG Graduate Thesis
Prereq: Permission of instructor
G (Fall, IAP, Spring, Summer)
Units arranged
Can be repeated for credit.

Program of research leading to the writing of a Ph.D. thesis; to be arranged by the student and an appropriate MIT faculty member.

Information: W. Minicozzi
18.S096 Special Subject in Mathematics
Prereq: Permission of instructor
U (Fall, Spring)
Units arranged
Can be repeated for credit.

Opportunity for group study of subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to departmental approval. 18.S097 is graded P/D/F.

Fall: A. Moitra, P. Parrilo. Spring: H. Cohn

18.S097 Special Subject in Mathematics
Prereq: Permission of instructor
U (IAP)
Units arranged [P/D/F]
Can be repeated for credit.

Opportunity for group study of subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to departmental approval. 18.S097 is graded P/D/F.

Information: W. Minicozzi

18.S190 Special Subject in Mathematics
Prereq: Permission of instructor
U (Fall, Spring; second half of term)
Units arranged
Can be repeated for credit.

Opportunity for group study of subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to departmental approval.

Staff

18.S191 Special Subject in Mathematics
Prereq: Permission of instructor
U (Fall)
Units arranged
Can be repeated for credit.

Opportunity for group study of subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to departmental approval.

Staff

18.S995 Special Subject in Mathematics
Prereq: Permission of instructor
G (Spring)
Units arranged
Can be repeated for credit.

Opportunity for group study of advanced subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to departmental approval.

Staff

18.S996 Special Subject in Mathematics
Prereq: Permission of instructor
G (Fall)
Units arranged
Can be repeated for credit.

Opportunity for group study of advanced subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to Departmental approval.

Fall: A. Harrow

18.S997 Special Subject in Mathematics
Prereq: Permission of instructor
G (IAP)
Units arranged
Can be repeated for credit.

Opportunity for group study of advanced subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to Departmental approval.

Staff

18.S998 Special Subject in Mathematics
Prereq: Permission of instructor
G (Spring)
Units arranged
Can be repeated for credit.

Opportunity for group study of advanced subjects in mathematics not otherwise included in the curriculum. Offerings are initiated by members of the Mathematics faculty on an ad hoc basis, subject to Departmental approval.

Staff